Mark schemes

Q1.

$3x2 - 6x + x - 2$
or $3x2 - 5x - 2$

4 terms with at least 3 correct

M1

$3x2 + (a - \text{their } 5)x - \text{their } 2 + b$
or a − their 5 = 8
or b − their 2 = − 5

M1

$a = 13$

A1

b = − 3

A1

Additional Guidance

a − their 5 = 8, $a = 13$

M1A1

a − their 5 = 8, a = 13 and $b - 2 = -5$, $b = -3$

M1A1M1A1

$13x - 3$

M1A1M1A1

[4]

Q2.

$x^2 + ax + ax + (a2)$

or $x2 + 2ax + (a2)$

or $2a = 8$ or $a2 + b = 7$

M1

$(x + 4)2$

or $a = 4$ or $b = -9$

A1

$(x + 4)2 - 9$

allow $a = 4$ and $b = -9$

A1

[3]

Q3.

$2(x + 5)^2$

B1

[1]

Q4.

$n^2 + (n + 1)^2$

Condone missing brackets if recovered

M1

$n^2 + n^2 + 2n + 1$

M1 dep

$2n^2 + 2n + 1$

A1

$2n(n + 1) + 1$

Accept $2n(n + 1) + 1 = 2n^2 + 2n + 1$ or $2n(n + 1) = 2n^2 + 2n$
for this mark provided the first 3 marks have been earned

A1

Complete solution with all stages clearly shown

Strand (ii)
Clear explanation
Do not award if first line assumes answer with use of = sign
eg $n^2 + (n + 1)^2 = 2n(n + 1) + 1$

Q1

Alternative method

$n^2 + (n + 1)^2 - 2n(n + 1)$

Condone missing brackets if recovered

M1

$n^2 + n^2 + 2n + 1 - 2n(n + 1)$

M1 dep

$2n^2 + 2n + 1 - 2n(n + 1)$

A1

$2n^2 + 2n + 1 - 2n^2 - 2n$

Allow $2n^2 + 2n + 1 - (2n^2 + 2n)$

A1

Complete solution with all stages clearly shown

Strand (ii)
Clear explanation
Do not award if first line assumes answer with use of = sign
eg $n^2 + (n + 1)^2 - 2n(n + 1) = 1$

Q1

[5]

Q5.

$\frac{n(n-1)+n(n+1)}{2}$

This mark is for combining fractions or if fractions dealt with separately, for combining n2 terms correctly

$\frac{n^2-n+n^2+n}{4}$ *is B0 as incorrect combining of fractions*

B1

$\frac{n^2-n+n^2+n}{2} = \frac{2n^2}{2}$

This mark is for eliminating −n and n either by showing by crossing or writing on same line and writing next line without them

$\frac{n^2}{2} - \frac{n}{2} + \frac{n^2}{2} + \frac{n}{2}$

B1

$\frac{2n^2}{2} = n^2$

This mark is for cancelling 2 top and bottom

$\frac{n^2}{2} + \frac{n^2}{2} = n^2$

B1

Alternative Method

$\frac{n}{2}((n-1)+(n+1))$

This mark is for factorising out a common factor.

$\frac{n}{4}(n-1+n+1)$ *is B0 as incorrect factorisation*

B1

$\frac{n}{2}(2n)$

This mark is for combining terms inside bracket correctly

B1

n^2

$1n^2$ is OK

B1

[3]

Q6.

(a) $6x^2 + 3x - 8x - 4$

Must have 4 terms shown or implied, including a quadratic term, two linear terms and a constant term. Could be in a grid from box method

Allow one sign or arithmetic error for M1

M1

$6x^2 - 5x - 4$

$kx^2 - 5x - 4$ or $6x^2 - 5x - k$ both imply M1

A1

(b) $(ax \pm c)(bx \pm d)$

$ab = 6$, $cd = 4$ or -4

$6x(x - 4) + (1)(x - 4)$

$x(6x + 1) - 4(6x + 1)$

M1

$(6x + 1)(x - 4)$

Ignore any subsequent attempt to solve once the correct factorisation seen

A1

[4]

Q7.

$2(cx + 5) + c$ or $2cx + 10 + c$

M1

their $2cx = 6x$ or their $2c = 6$
or $c = 3$

Must have attempted fg(x)

M1

13

SC2 for 11

A1

[3]

Q8.

$2y^3 - 10y^2 + 4y - 3y^2 + 15y - 6$

Must have at least five terms with at least four correct

M1

$2y^3 - 10y^2 + 4y - 3y^2 + 15y - 6$

A1

$2y^3 - 13y^2 + 19y - 6$

ft from M1 A0

A1ft

[3]

Q9.

Alternative method 1

$4x^2 + 6xy + 6xy + 9y^2$

oe Allow one error

Implied by $4x^2 + 12xy + \ldots$ or $\ldots + 12xy + 9y^2$

M1

$4x^2 + 6xy + 6xy + 9y^2$ or $4x^2 + 12xy + 9y^2$

oe *Fully correct*

A1

$4x^3 + 6x^2y + 6x^2y + 9xy^2$

or $4x^3 + 12x^2y + 9xy^2$

or $-16x^2 - 24xy - 24xy - 36y^2$

or $-16x^2 - 48xy - 36y^2$

oe

ft correct multiplication of their expansion by x or by −4 if their expansion for first M1 has at least 3 terms after simplification

M1dep

$4x^3 + 12x^2y + 9xy^2 - 16x^2 - 48xy - 36y^2$

ft M1A0M1 if their first expansion has at least 3 terms after simplification

A1ft

Alternative method 2

$2x^2 + 3xy - 8x - 12y$

oe *Allow one error*

eg $2x^2 + 3xy - 8x + 12y$

M1

$2x^2 + 3xy - 8x - 12y$

oe *Fully correct*

A1

$4x^3 + 6x^2y - 16x^2 - 24xy$ or (+) $6x^2y + 9xy^2 - 24xy - 36y^2$

oe ft correct multiplication of their expansion by 2x or by 3y if their expansion for first M1 has at least 3 terms after simplification

M1dep

$4x^3 + 12x^2y + 9xy^2 - 16x^2 - 48xy - 36y^2$

ft M1A0M1 if their first expansion has at least 3 terms after simplification

A1ft

Additional Guidance

Terms and variables may be in any order for M and A marks

For M1 A1 M1dep terms may be seen in a grid

$4x^3 - 16x^2 + 9xy^2 - 36y^2$ from $(x - 4)(4x^2 + 9y^2)$

M0A0M0A0

In alt 2, condone $(2y^2 + 3xy - 8x - 12y)^2$ for M1A1 only

One error can be one incorrect term or a missing or extra term

Do not ignore fw when awarding the final A mark

If $(x - 4)(2x + 3y)$ and $(2x + 3y)2$ are both attempted and no answer is given, mark b
and award the better mark

[4]

Q10.

$a = 4$ or $(3x - 1)(4x + b)$

B1

$3ax2 + 3bx - ax - b$

or $3b - a = -19$

or $12x2 + 3bx - 4x - b$

M1

$3bx - 4x =$

$-19x$ or $3b - 4$

$= -19$ or $3b = -15$ or $b = -5$

or $(3x - 1)(4x - 5)$

This mark implies B1M2

M1

$a = 4$ and $b = -5$ and $c = 5$

A1

Additional Guidance

$3ax^2 + 3bx - 1ax - b$ or $3ax^2 + 3bx - ax - 1b$

M1

Condone $3x2a$ and $3xb$ and xa

[4]

Q11.

$(3a - b)(3a + b)$

B1 $(3a - b)(3a - b)$ or $(3a + b)(3a + b)$
or $(3a - b)2$ or $(3a + b)2$
or $(9a + b)(a - b)$ or $(9a - b)(a + b)$

B2

Additional Guidance

$(3a - b) \times (3a + b)$

B1

[2]

Q12.

(a) $3y(3y^2 - 2)$ or $-3y(2 - 3y^2)$

B1 3(3y3 – 2y) or y(9y2 – 6)
or –3(2y – 3y3) or –y(6 – 9y2) — B2

Additional Guidance

$3y(3y2 - 2)$ or $-3y(2 - 3y2)$ followed by incorrect further work

eg $3y(3y2 - 2) = 3y2(3y - 2)$ — B1

$3y(3y^2 - 2) = 3y(\sqrt{3}y + 2)(\sqrt{3}y - 2)$ — B2

$3y(3y^2 - 2) = 9y^3 - 6y$ (checking) — B2

$3y \times (3y^2 - 2)$ — B2

$3 \times (3y^3 - 2y)$ — B1

$y3(3y^2 - 2)$ — B1

(b) $(3x - 1)(x - 7)$ or $(1 - 3x)(7 - x)$

B1 $(3x + a)(x + b)$
where $ab = 7$ or $a + 3b = -22$
or $(a - 3x)(b - x)$
where $ab = 7$ or $a + 3b = 22$ — B2

Additional Guidance

$(3x + 1)(x + 7)$ — B1

$(3x - 1)(x - 7)$ — B1

$(3x - 4)(x - 6)$ — B1

$(7 - 3x)(1 - x)$ — B1

$(10 - 3x)(4 - x)$ — B1

$(3x - 1) \times (x - 7)$ — B2

Ignore any 'solutions' seen

eg $(3x - 1)(x - 7)$ in working with $\frac{1}{3}$ and 7 on answer line

B2

[4]

Q13.

$(t + 4)(t^2 + 4t + 4t + 16)$

oe Must be correct

M1

$t^3 + 4t2 + 4t2 + 16t + 4t2 + 16t + 16t + 64$

ft From their $(t + 4)(t^2 + 4t + 4t + 16)$

oe Must have at least 4 terms correct

M2 t3 + 3t2(4) + 3t (4)2 + 43 oe

M1

$t^3 + 12t2 + 48t + 64$

A1

[3]

Q14.

Alternative method 1 – completing the square

$(x + \frac{1}{2})^2 + \ldots$

M1

$(x + \frac{1}{2})^2 - (\frac{1}{2})^2 + 1$

or $(x + \frac{1}{2})^2 - \frac{1}{4} + 1$

or $(x + \frac{1}{2})^2 + \frac{3}{4}$

oe

A1

$(x + \frac{1}{2})^2 \geq 0$ and $\frac{3}{4} > 0$ and always positive

oe

A1

Alternative method 2 – real roots

$\frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times 1}}{2 \times 1}$

or a correct sketch showing a quadratic curve with turning point above the x-axis

oe

M1

States no values on x-axis

oe

A1

States no values on x-axis and (minimum value =) $\frac{3}{4}$

oe

A1

Alternative method 3 – Calculus

$2x + 1 = 0$

M1

$x = -\frac{1}{2}$

A1

(minimum value =) $\frac{3}{4}$

A1

Alternative method 4 – Explanation method

If $x \geq 0$, $x^2 \geq 0$ and $x \geq 0$ $(1 > 0)$ so $x^2 + x + 1 > 0$

and If $-1 < x < 0$ $x^2 > 0$ and $x + 1 > 0$ so $x^2 + x + 1 > 0$

and If $x \leq -1$ $x^2 > x$ and $x^2 + x > 0$ so $x^2 + x + 1 > 0$

Accept $x > 0$ for $x \geq 0$
B2 for two correct statements
B1 for one correct statement

B3

Additional Guidance

Calculating pairs of coordinates alone

M0A0A0

[3]

Q15.

(a) $(x - 5)^2 + 1$

M1

$x^2 - 5x - 5x + 25 + 1$
$= x^2 - 10x + 26$

A1

(b) $x^2 + 1 - 5$ or $x^2 - 4$

B1

$x^2 - 10x + 26 =$ their $(x^2 - 4)$

M1

$-10x = -4 - 26$
or $-10x = -30$
or $10x = 30$

oe

M1

3

A1

Q16.

$8 \times \frac{1}{2} n(n + 1)$ (+1)

M1

$4n(n + 1)$ (+ 1)

or $4n2 + 4n$(+ 1)

M1dep

$(2n + 1)2$ or $(2n + 1)(2n + 1)$

A1

$(2n + 1)2$ is a square number

oe

or $2n + 1$ is odd

and odd × odd = odd

odd^2 = odd

or multiple of 4 is even

and even + 1 = odd

or

n(n + 1) is odd × even or even × odd

so n(n + 1) is even

or $4(n2 + n)$ is even

and even + 1 = odd

and even × 4 = even

and even + 1 = odd

or $4n2$ is even and $4n$ is even

and even + 1 = odd

A1

$(2n + 1)2$ is a square number
and

or $2n + 1$ is odd

and odd × odd = odd

Strand (ii)

Both parts of the proof required.

or multiple of 4 is even

and even + 1 = odd

or

n(n + 1) is odd × even or even × odd

so n(n + 1) is even

or 4(n2 + n) is even

and even + 1 = odd

and even × 4 = even

and even + 1 = odd

or 4n2 is even and 4n is even

and even + 1 = odd

SC1 for 8 × S = even

and even + 1 = odd

Q1

[5]

Q17.

5f(x) = 4x − 3 or 5f(x) + 3x = 4

or 5y = 4x − 3 or 5 y + 3x = 4

or 5x = 4y − 3 or 5x + 3y = 4

Accept any letter used for y

M1

$\frac{5f(x) + 3}{4}$ (= x)

or $\frac{5y + 3}{4}$ (= x)

M1

$\frac{5x + 3}{4}$

Condone y = (or any other letter)

A1

[3]

Q18.

Full explanation stating

one of a + b or a – b must be 1

and

a + b cannot be 1

and

a – b must be 1

B1 partial explanation

ie a + b or a – b must be 1

or

a + b cannot be 1
or
a – b must be 1

B2

[2]

Q19.

(a) $3(x + 2)(x - 2)$

B1 for $3(x^2 - 4)$ or $(3x + 6)(x - 2)$ or $(x + 2)(3x - 6)$

B2

(b) $(5x + ay)(x + by)$

where $ab = \pm 12$ or $a + 5b = \pm 4$

M1

$(5x \pm 6y)(x \pm 2y)$

for correct y terms in correct brackets, but with a sign error

A1

$(5x - 6y)(x + 2y)$

A1

[5]

Q20.

(a) $5(m + 2p)(m - 2p)$

B2 $(5m + 10p)(m - 2p)$ or $(5m - 10p)(m + 2p)$

B1 $5(m^2 - 4p^2)$ or $(5m + ap)(m + bp)$ where $ab = \pm 20$

B3

(b) Their $(m + 2p) = 0$ or

Their $(m - 2p) = 0$

oe e.g. $m = -2p$ or $m = 2p$

May substitute for p at this stage

M1

−30 and 30

A1

Alternative method

$5m^2 - 20 \times 15 \times 15 = 0$

oe e.g. $5m^2 = 4500$

M1

−30 and 30

A1

[5]

Q21.

$\frac{x}{3}$

B1

[1]

Q22.

$5n^2 - 5n + 3n - 3$

oe 4 terms with 3 correct including a term in n^2

M1

$5n^2 - 5n + 3n - 3$

Fully correct

oe e.g. $5n^2 - 2n - 3$

A1

$6n^2 - 3$

A1

$3(2n^2 - 1)$ or states that both terms are multiples of 3

oe

A1

[4]

Q23.

$(3x + a)(x + b)$

where $ab = 8$ or $a + 3b = 14$

or

$3x(x + 4) + 2(x + 4)$

or

$x(3x + 2) + 4(3x + 2)$

M1

$(3x + 2)(x + 4)$

oe

A1

[2]

Q24.

Alternative method 1

$(w =)\ x - 2$ and $(y =)\ x + 2$

Allow $(x =)\ w + 2$ and $(x =)\ y - 2$

M1

$(x - 2)(x + 2) + 4$

or

$wy = (x - 2)(x + 2)$ and $wy = x^2 - 4$

M1

$= x^2 - 4 + 4$
and $x^2 - 4 + 4 = x^2$

All steps must be seen
SC1 correct numerical example with all steps shown

A1

Alternative method 2

$(x =)\ w + 2$ and $(y =)\ w + 4$

Allow $(x =)\ w + 2$ *and* $(x =)\ y - 2$

M1

$(w)(w + 4) + 4$

M1

$= w^2 + 4w + 4$
and $w^2 + 4w + 4 = (w + 2)^2$
and $(w + 2)^2 = x^2$

All steps must be seen
SC1 correct numerical example with all steps shown

A1

Alternative method 3

$(x =)\ y - 2$ and $(w =)\ y - 4$

Allow $(x =)\ w + 2$ *and* $(x =)\ y - 2$

M1

$(y)(y - 4) + 4$

M1

$= y^2 - 4y + 4$
and $y^2 - 4y + 4 = (y - 2)^2$
and $(y - 2)^2 = x^2$

All steps must be seen
SC1 correct numerical example with all steps shown

A1

Additional Guidance

$x = 3$, $w = 1$, $y = 5$ and $1 \times 5 + 4 = 9$

0

$x = 3$, $w = 1$, $y = 5$ and $1 \times 5 + 4 = 9$ and $9 = 3^2$

SC1

$1 \times 5 + 4 = 9$ and $9 = 3^2$

0

[3]